Solid-state electronics

Fermi-Dirac Distribution and Fermi Level

David Collins
C10736685
DT021 Year 2
Dublin Institute of Technology

April 2012

Abstract

This report examines the Fermi-Dirac distribution and Fermi Level as they relate to semiconductor materials. The Fermi-Dirac distribution is central to understanding electron and hole distributions in energy bands.

1 Introduction

1.1 What is the Fermi-Dirac Distribution?

The Fermi-Dirac distribution is a *probability* distribution given by the formula

$$f_F(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \tag{1}$$

In the context of a semiconductor crystal, the distribution gives the probability that a state at a given energy will be occupied by an electron.

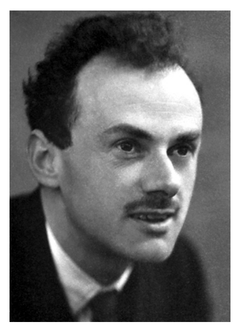


Figure 1: Paul Dirac — one of the founders of quantum theory

1.1.1 Properties

We can note the following properties of the Fermi-Dirac distribution.

1. As a probability, its value is between zero and one. I.e. $0 \leqslant f_F(E) \leqslant 1$.
2. For energies much greater than the Fermi energy, the Fermi-Dirac distribution approximates to the Maxwell-Boltzmann formula. I.e. $f_F(E) \approx f_F(E)|_{MB} \forall E \gg E_F$.
3. At the Fermi energy itself, the distribution evaluates to a half. $f_F(E_F) = \frac{1}{2}$.

The Maxwell-Boltzmann approximation is

$$f_F(E)|_{MB} = e^{-(E-E_F)/kT}. \tag{2}$$

1.2 What is the Fermi level?

The Fermi level is an energy level for which the probability of occupation by an electron (or hole) is equal to one half.

$$E_F = E : f_F(E) = \frac{1}{2}$$

Alternatively, it can be defined as the energy level above which no states are occupied at absolute zero temperature.

1.3 Charge-carrier concentration

Crucially, the actual concentration of charge carriers is the *product* of the density of states and the Fermi-Dirac distribution. In other words, the concentration of carriers is the number of states per unit volume per unit energy times the probability that these states are occupied.

$$N(E) = f_F(E)g(E) \tag{3}$$

1.4 Why are these parameters important?

The Fermi-Level and Fermi-Dirac distribution are fundamentally important because — given the concentration of impurity elements — they can be used to determine the concentrations of charge carriers. These, in turn, help us to calculate the charge density — and hence how the semiconductor will respond when subject to a certain electric field.

The primary appeal of semiconductor materials is that they allow electricity to be easily controlled. Using the Fermi-Dirac distribution is central to understanding how this is done.

2 Effects of doping concentration

The electron concentration in the conduction band is

$$n_0 = N_c e^{-(E_c-E_F)/kT}. \tag{4}$$

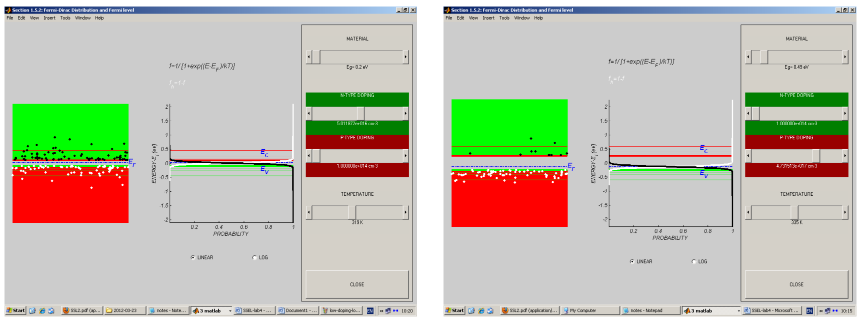
The difference term $E_c - E_F$ depends on the donor and acceptor impurity concentrations according to

$$E_c - E_F = kT \ln\left(\frac{N_c}{N_d}\right). \tag{5}$$

We note two relationships here:

1. the conduction-band electron concentration depends on the difference between the conduction band energy and the Fermi energy;
2. the Fermi energy level, in turn, depends on the ratio of the effective density of states to the donor concentration.

Hence, as expected, the number of electrons available for conduction varies with the level of impurity doping.



(a) High donor impurity level

(b) High acceptor impurity level

Figure 2: Effects of doping concentrations

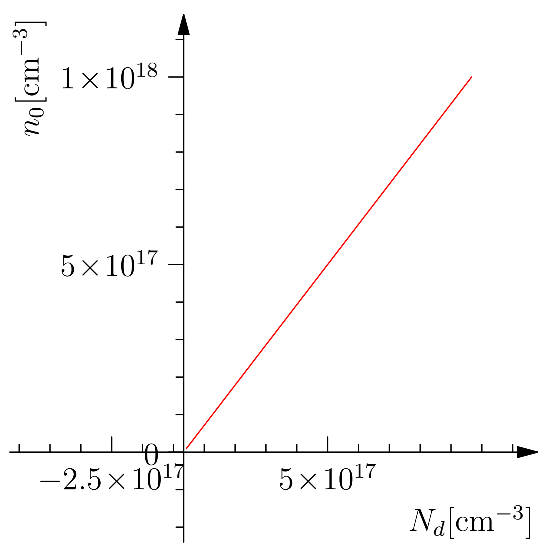


Figure 3: Concentration of electrons for an n-type semiconductor

For an n-type semiconductor, the concentration of electrons in the conduction band is approximately equal to the concentration of donor impurity elements.

$$\boxed{n_0 \approx N_d}$$

Intuitively, this makes sense. Each atom of a group V element that is added to the crystal introduces an extra electron to the structure. We need to consider the effects of temperature to understand this properly, however. This is discussed below.

3 Effects of Temperature

We might expect that, for a given concentration, the number of electrons in the conduction energy band will increase with temperature. This is indeed the case.

3.1 Temperature and the Fermi-Dirac distribution

Referring to equation 1, we see that the Fermi-Dirac distribution has a direct dependence on temperature. This is illustrated in Figure 4, and can be observed by comparing Figures 5 and 6 also. At absolute zero, the probability of a state

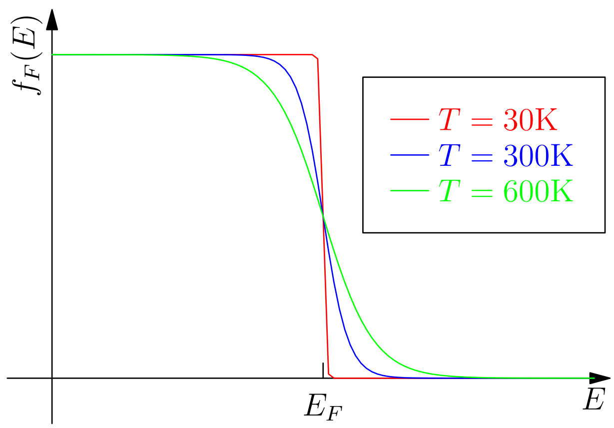


Figure 4: Variation in the Fermi-Dirac distribution with temperature

in the conduction band being occupied is zero; no electrons will be available for conduction. As the temperature increases, so does the probability of energy states above the Fermi Energy being occupied. Note, in Figure 5, that the Fermi-Dirac distribution curve is a different shape to that in Figure 6.

3.2 Ionization

The donor energy level is actually slightly below that of the conduction band. At absolute zero temperature, all donor electrons are bound to their impurity atoms. As the temperature increases, however, some of these electrons break free of their parent atoms and join the conduction band. At room temperature,

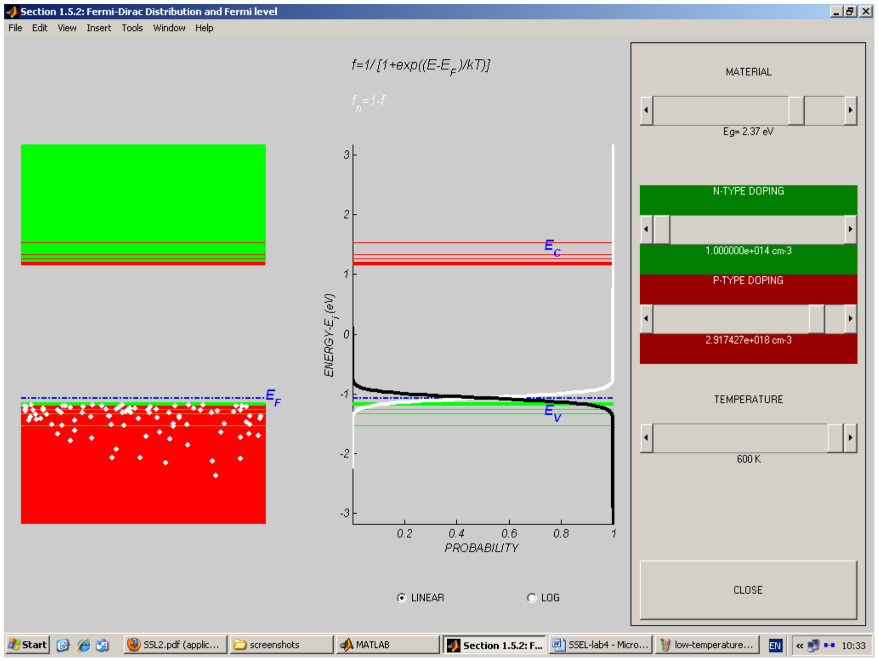


Figure 5: The Fermi-Dirac probability distribution changes at higher temperature

we almost have *complete ionization*; essentially all of the electrons in the donor level have moved to the conduction band. Hence, $n_o \approx N_d$.

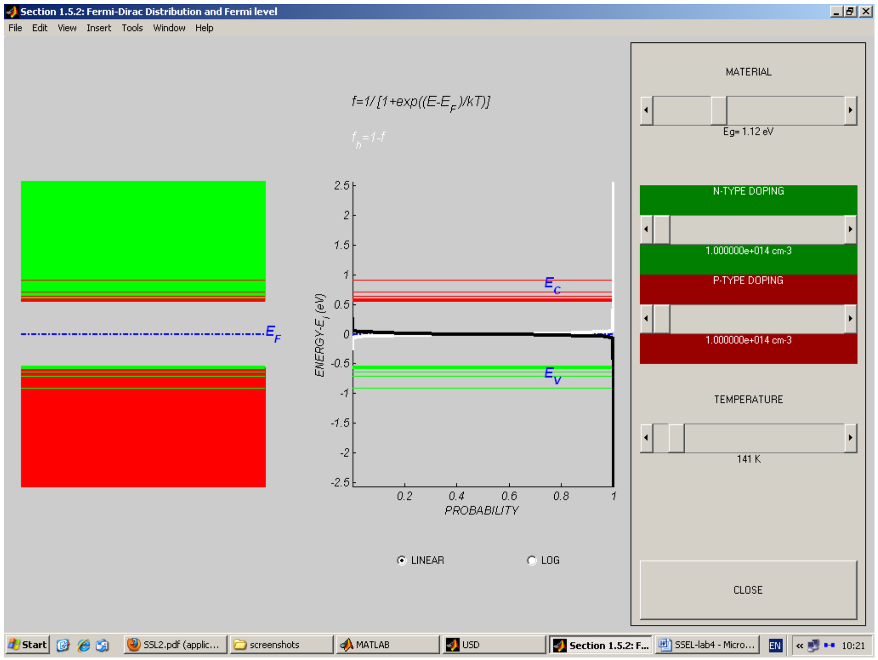


Figure 6: At lower temperature less charge carriers are available for conduction

4 Summary

This report has highlighted the following:

- how the levels of donor and acceptor impurity elements affect the electron and hole concentrations,
- the concentration of both holes and electrons increases with temperature, and
- how the Fermi level is affected by both temperature and impurity levels.

We have confirmed these relationships either by using the mathematical formulae or by computer simulation — or both.

The value of these insights lies — at least partially — in enabling an engineer to subsequently determine macro-parameters of a semiconductor. These parameters include the conductivity and current density of a device, for example. Hence, the Fermi-Dirac distribution provides a link between non-intuitive quantum systems on one hand, and everyday electronic devices on the other.